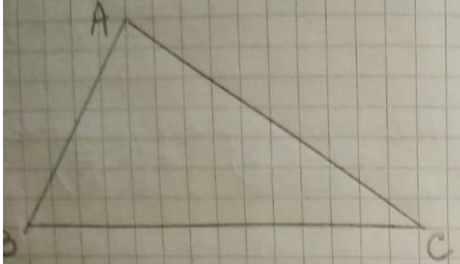
Reciproca teoremei lui Pitagora

- Dacă într-un triunghi, pătratul lungimii unei laturi este egal cu suma pătratelor lungimilor celorlalte două laturi, atunci triunghiul este dreptunghic.



Dacă $BC^2 = AB^2 + AC^2 \Rightarrow \sphericalangle BAC = 90^\circ$

Așadar BC = ipotenuza

AB, AC = catete

Probleme rezolvate – culegere, fișa 7

1. a) ΔABC

$AB = 3$ cm

$AC = 4$ cm

$BC = 5$ cm

Verificați dacă ΔABC este dreptunghic

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$ (adevărat) $\Rightarrow$

$\Rightarrow BC^2 = AB^2 + AC^2 \Rightarrow \sphericalangle BAC = 90^\circ$

$\Rightarrow \Delta ABC$ este dreptunghic.

b) ΔABC

$AB = 4\sqrt{3}$ cm

$AC = 4$ cm

$BC = 4\sqrt{6}$ cm

Verificați dacă ΔABC este dreptunghic

$(4\sqrt{6})^2 = (4\sqrt{3})^2 + 4^2$

$16 \cdot 6 = 16 \cdot 3 + 16$

$96 = 48 + 16$

$96 = 64$ (fals) $\Rightarrow \sphericalangle BAC \neq 90^\circ$

$\Rightarrow \Delta ABC$ nu este dreptunghic.

(2) ΔMNP

$(MN, NP, MP) \sim (15, 36, 39)$

Verificați dacă ΔMNP este dreptunghic

$(MN, NP, MP) \sim (15, 36, 39) \Rightarrow \frac{MN}{15} = \frac{NP}{36} = \frac{MP}{39} = k$

$\Rightarrow MN = 15 \cdot k$

$NP = 36 \cdot k$

$MP = 39 \cdot k$

$(39k)^2 = (15 \cdot k)^2 + (36 \cdot k)^2$

$1521 \cdot k^2 = 225 \cdot k^2 + 1296 \cdot k^2$

$1521 \cdot k^2 = k^2 \cdot (225 + 1296)$

$1521 \cdot k^2 = k^2 \cdot 1521$ (adevărat) $\Rightarrow \sphericalangle MNP = 90^\circ \Rightarrow$

$\Rightarrow \Delta MNP$ dreptunghic

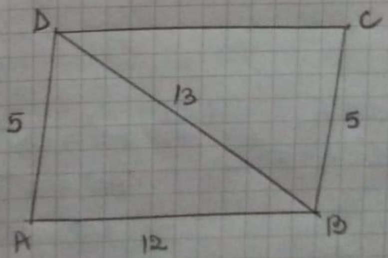
(3) ABCD paralelogram

$AB = 12$ cm

$BC = 5$ cm

$BD = 13$ cm

ABCD dreptunghi



ABCD paralelogram $\Rightarrow AD = BC = 5$

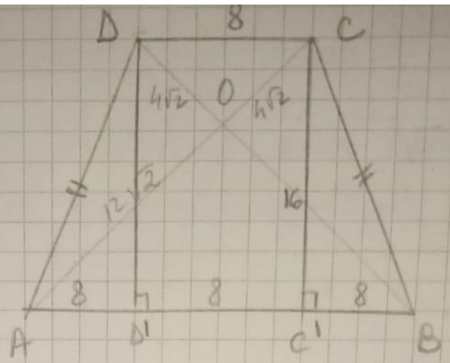
ΔABD: $13^2 = 12^2 + 5^2$

$169 = 144 + 25$

$169 = 169$ (adevărat) $\Rightarrow BD^2 = AB^2 + AD^2 \Rightarrow$

$\left.\begin{array}{l}\Rightarrow \sphericalangle DAB = 90^\circ \\ \text{ABCD paralelogram}\end{array}\right\} \Rightarrow$ ABCD dreptunghi

(4) ABCD trapez isoscel



$AB \parallel CD$

$AB = 24$ cm

$CD = 8$ cm

~~$AA' \perp$~~ $CC' \perp AB$

$CC' = 16$ cm

(I)

$AC \perp BD$

(II)

Constr. $\left.\begin{array}{l} DD' \perp AB \\ CC' \perp AB \end{array}\right\} \Rightarrow \left.\begin{array}{l} CC' \parallel DD' \\ DC \parallel D'C' \\ \sphericalangle DD'C' = 90^\circ \end{array}\right\} \Rightarrow DCC'D'$ dreptunghi $\Rightarrow D'C' = DC = 8$ cm

$CC' = DD' = 16$ cm

$\triangle DD'A \equiv \triangle CC'B$ (I.C.) $\Rightarrow AD' = C'B = (24-8):2 = 8$ cm

Constr. $AC \cap BD = \{O\}$

ABCD trapez isoscel $\Rightarrow AC = BD$

$OA = OB$

$OC = OD$

$\left.\begin{array}{l} \triangle CC'A \\ \sphericalangle CC'A = 90^\circ \end{array}\right\} \xRightarrow{T.Pit.} AC^2 = C'C^2 + C'A^2$

$AC^2 = 16^2 + 16^2$

$AC^2 = 2 \cdot 16^2$

$AC = \sqrt{2 \cdot 16^2}$

$AC = 16\sqrt{2}$ cm

$DC \parallel AB \xRightarrow{t.f.a.} \triangle COD \sim \triangle AOB \Rightarrow \frac{CO}{AO} = \frac{DO}{BO} = \frac{CD}{AB}$

$\frac{CO}{AO} = \frac{8^{(8}}{24} \Rightarrow \frac{CO}{AO} = \frac{1}{3} \Rightarrow$

$\left.\begin{array}{l} \Rightarrow AO = 3 \cdot CO \\ AO + CO = 16\sqrt{2} \end{array}\right\} \Rightarrow 3 \cdot CO + CO = 16\sqrt{2}$

$4 \cdot CO = 16\sqrt{2}$

$CO = 4\sqrt{2}$ cm $\Rightarrow AO = 12\sqrt{2}$ cm

ΔAOB:

$OA = OB = 12\sqrt{2}$ cm

$AB = 24$ cm

$\Rightarrow$

$24^2 = (12\sqrt{2})^2 + (12\sqrt{2})^2$

$24^2 = 12^2 \cdot 2 + 12^2 \cdot 2$

$24^2 = 12^2 \cdot (2+2)$

$24^2 = 12^2 \cdot 4$

$576 = 144 \cdot 4$

$576 = 576$ (adevărat) $\Rightarrow AB^2 = OA^2 + OB^2$

$\Rightarrow \measuredangle AOB = 90^\circ \Rightarrow AC \perp BD$.

Obs. Se poate aplica reciproca teoremei lui Pitagora în oricare dintre triunghiurile: ΔBOC, ΔCOD, ΔDOA.

Temă acasă: Fișa 7 pr. 1(c, d, e), 5, 6

Tema 7 pr. 1, 2, 3, 4.